Hale School

Mathematics Specialist

Test 1 --- Term 1 2018

Complex Numbers

/ 44

Name: ______________________________

Instructions:

- **Calculators are NOT allowed**
- **External notes are not allowed**
- **Duration of test: 45 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

All arguments must be given using principal values.

1. **[3, 4 = 7 marks]**

Give exact expressions for each of the following in the form $a + bi$:

(a) $\dfrac{\overline{1-i}}{(2+i)^2}$

(b) $\left(1-\sqrt{3}i\right)^5$

2. **[4, 2 = 6 marks]**

(a) Use de Moivre's theorem to find all the exact solutions to the equation

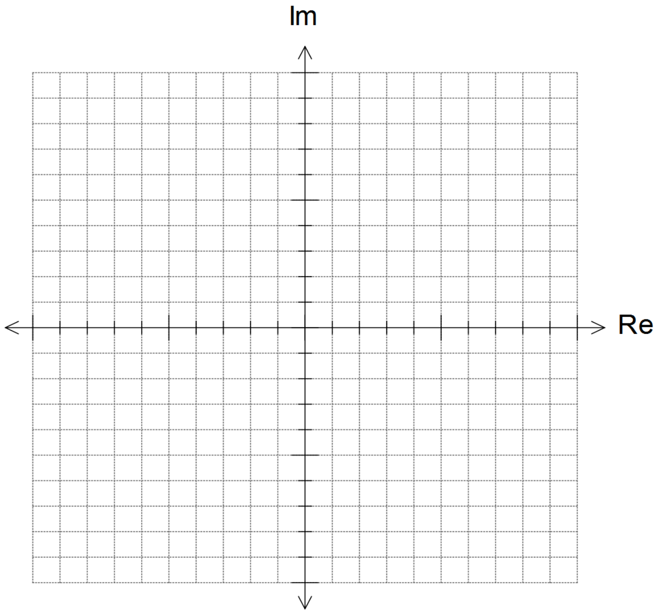
$z^4 = i$ in polar form

(b) Suppose α and β are two distinct roots of $z^n = i$, where n is a positive integer. Explain why $|\alpha + \beta| < 2$.

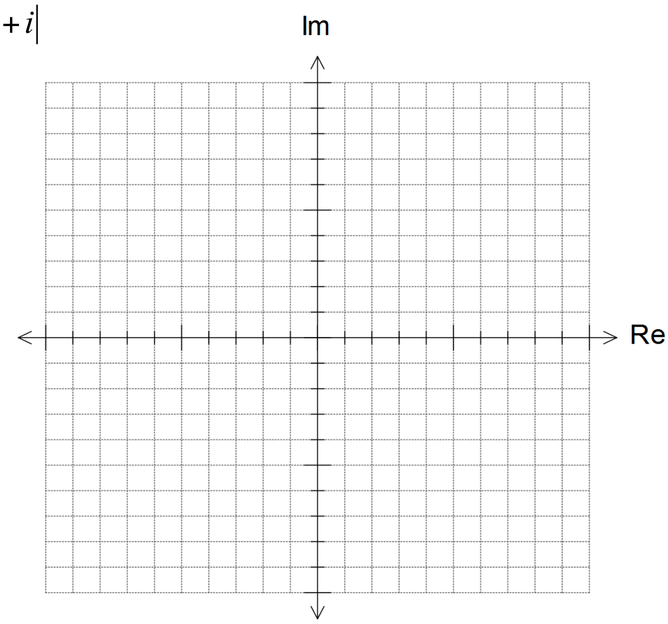
3. **[3, 3 = 6 marks]**

Sketch the following loci on the complex planes provided:

(a) $\text{Re}(iz+1)=\text{Im}(z)$



(b) $|z-(1+3i)|\le|z+1+i|$



4. **[4 marks]**

Given $z = (2a + 3i)^3$ and $a \in R^+$, find the value(s) of a such that $\arg z = 135°$.

5. **[4 marks]**

Describe the locus of points defined by the equation $|z - i| = 2|z + 1|$.

6. **[4, 4 = 8 marks]**

(a) The polynomial $x^3 + ax + b$ has a factor of $x+2$ and a remainder of -60 when divided by $x-2$. Determine the values of a and b.

(b) One root of $P(z) = z^3 + az^2 + 3z + 9$ is purely imaginary. If a is real, find a and hence factorise $P(z)$ into linear factors.

7. [4, 5 = 9 marks]

Show that if $z = cis\theta$ then:

a) $z^n - \dfrac{1}{z^n} = 2i\sin n\theta$

b) Use the previous result to show that $\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta$

_____End of Test_____